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PROJECT ECHO- 961-Mc LOWER-SIDEBAND UP-CONVERTER FOR SATELLITE-TRACKING RADAR

M. Uenohara and H. Seidel
Bell Telephone Laboratories

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SUMMARY

A 961-Mc lower-sideband up-converter was specially designed to serve as preamplifier for the satellite-tracking radar used in Project Echo. The amplifier and its power supply are separately boxed and are installed directly behind the tracking antenna. The amplifier has been functioning most satisfactorily and has been used in routine manner to track the Echo satellite from horizon to horizon. This paper describes the design considerations, and details the special steps taken to ensure that the amplifier met the particular system needs of low noise, absolute stability, insensitivity to temperature fluctuations, and high input-power level before the onset of gain compression. The satisfactory operation of this amplifier confirms the great potentiality of parametric amplifiers as stable, low-noise, high-frequency receivers.

PREFACE

The Project Echo communications experiment was a joint operation by the Goddard Space Flight Center of the National Aeronautics and Space Administration (NASA), the Jet Propulsion Laboratory (JPL), the Naval Research Laboratory (NRL), and the Bell Telephone Laboratories (BTL). The equipment described herein, although designed by BTL as part of its own research and development program, was operated in connection with Project Echo under contract NASW-110 for NASA. Overall technical management of Project Echo was the responsibility of NASA's Goddard Space Flight Center.

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INTRODUCTION

The satellite-tracking radar amplifier operates at a center-band frequency of 961 Mc in a lower-sideband mode and is pumped at 11.7 kMc; the output frequency of the amplifier is 10.739 kMc. The amplifier is unconditionally stable and its gain remains constant over long periods. Its overall noise figure, including the following mixer and IF stage, is less than 1.6 db; the gain at the center band is 22 db and the bandwidth is 20 Mc. The input power level that reduces the amplifier gain by 1 db is -26 dbm.

Since lower-sideband operation is employed, the amplifier is a regenerative one and basically very sensitive to changes in circuit impedance and to fluctuations in the power supply. This feature has often been considered a disadvantage of parametric amplifiers in systems applications. To allay such objections, special precautions have been taken to stabilize the parametric amplifier. In particular, good isolators are provided to isolate the amplifier from any impedance fluctuations produced by antenna mismatches. Any fluctuation in the pump frequency is continuously corrected by an automatic frequency control, and all components, especially those near the diode, are made extremely rigid, thereby eliminating the possibility of impedance variations caused by mechanical vibrations. It is only through such care that the regenerative device can be and indeed has been made a stable and reliable amplifier (cf. Reference 1).

The system requirements of Project Echo made it necessary to pay special attention to the following two points:

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(1) The amplifier had to be insensitive to large environmental changes; this was necessary because the amplifier was installed directly behind the antenna in order to reduce cabling losses. Otherwise, the insertion loss of the input circuit would degrade the noise performance of the system.

(2) The gain-compression characteristics had to be sufficiently good that, with an input power level of -26 dbm, the gain would be still within 1 db of the small-signal gain; at the same time, the overall system noise figure had to be less than 1.8 db. This compression specification arose because the maximum possible leakage power from the 960-Mc satellite communications transmitter was about -26 dbm at the radar receiver station.

It is difficult to construct a filter which has a low insertion loss at 961 Mc while suppressing a frequency only 1 Mc away. If the gain compression were severe, with an input level of -26 dbm, the amplifier would be saturated by the leakage signal and the radar signal could not be detected. The power level at which compression sets in could be increased simply by decreasing the gain of the amplifier.* However, one must pay a penalty for decreasing the amplifier gain, since the overall system noise figure increases because of the increased noise contribution of the mixer and IF stages. The compression problem is thus not optimally solved simply by decreasing the amplifier gain; it requires a careful evaluation of the circuit design parameters.

The purpose of this paper is to describe the design consideration and the performance of the 961-Mc amplifier, with special emphasis on the precautions taken to ensure that this amplifier met all the system requirements.

SPECIFICATIONS

The specifications for the 961-Mc preamplifier used with the Project Echo satellite-tracking radar are given in Table 1.

Table 1
961-Mc Preamplifier Specifications

Parameter	Specification
Center-band frequency	961 Mc,
Overall system noise figure	< 1.8 db,
Input power level at 960 Mc for a gain compression of 1 db	> -26 dbm.

*For this particular amplifier the gain is almost constant until the output power reaches about -3 dbm. For example, for a gain of 22 db, the level at which compression sets in is about -25 dbm.

The amplifier had to be unconditionally stable, providing constant gain for minimum periods of one hour. The amplifier also had to be installed as close as possible to the antenna to reduce the cabling loss from the antenna to the amplifier.

According to these specifications the amplifier was to be packed in a small box which would be mounted on the back of the parabolic dish. It was expected that the temperature of the amplifier's environment would vary 80°F between midsummer and midwinter, and the performance of the amplifier had to be insensitive to such large environmental change. The amplifier had also to be insensitive to mechanical vibrations.

SELECTION OF THE MODE OF OPERATION

There are three presently accepted practical modes of operation of variable reactance communication amplifiers. They are, respectively, the upper-sideband, the lower-sideband, and the reflection types. All are nondegenerate. If ω_s is the signal frequency and ω_x is either the upper- or lower-sideband frequency, corresponding to either a positive or negative resistance device, then the minimum achievable system noise figures for uncooled systems are:

$$F = 1 + \frac{\omega_s}{\omega_x} \frac{T_L}{290} \text{ (upper-sideband);} \quad (1)$$

$$F = 1 + \frac{\omega_s}{\omega_x} \text{ (lower-sideband or reflection),} \quad (2)$$

where T_L is the effective temperature of the load, a fictitious quantity representing second-stage noise through the relationship

$$T_L = (F_2 - 1) 290, \quad (3)$$

where F_2 is the noise figure of the second stage.

The above formulas assume that the varactors are totally noise-free and, further, that the negative-resistance modes may be operated at infinite gain. While neither assumption is to be taken too seriously, they permit, nevertheless, elimination of the upper-sideband device from major consideration. The load temperature T_L may be optimistically estimated to be 7 db for a mixer IF combination. Assuming a probable value of ω_s/ω_x of the order of one-tenth, then with no other consideration of noise sources, this term would account for about 1-1/2 db in its own right.

Both the lower-sideband and the reflection types of amplifier system have minimum excess temperatures when operating at an infinite gain condition. They provide identical

noise figures and, at this gain, only the diode noise terms enter into account. This condition of infinite gain is evidently a singular one and corresponds, in principle, to the situation in which the diode regenerates its own losses as well as those of the idler-frequency cavity. Viewed back from the idler port, the amplifier corresponds to a zero impedance source. The idler load must correspondingly be decoupled infinitely if power is to be drawn from that source. It is little wonder, in the case of infinite gain, that the regeneration requirements of the lower-sideband amplifier demand a higher order of infinity of regeneration than does the reflection type of device, and that it is by far the more unstable of the two.

As we drop down from the infinite gain condition, the negative source impedance rises from zero and the coupling coefficient to the load likewise rises from zero. The details of the noise sources within the load then become increasingly significant in determining overall performance. The stability situation is ultimately reversed, in that the frequency-ratio gain available in the lower-sideband converter, in accounting for the second-stage noise, diminishes the need for regenerative gain compared to the reflection device, and the lower-sideband amplifier becomes more suitable for operation.

There were four reasons why a moderate gain was desired in contrast to a very high gain. These eventually made the lower-sideband converter the desired choice in system considerations for the Echo radar. They were:

- (1) A reasonably wide bandwidth was desired to avoid frequency stability problems;
- (2) Too high a gain leads to compression problems;
- (3) Very large gain leads to pump stability problems;
- (4) Antenna mismatches require excessive isolation at high regenerative gain and noise is introduced through isolator forward loss.

The choice between the reflection and the lower-sideband amplifier depended on the penalty paid to system noise performance for finite gain operation. The total system temperature is

$$T_{\text{sys}} = T_{\text{amp}} (G) + \frac{T_L}{G} \quad (4)$$

In either amplifier type, T_L is typically of the order of 1200°K, and a gain of 20 db is thoroughly adequate to reduce second-stage noise. It may be shown that T_{amp} is well behaved as G decreases from infinity, and that the increase in system noise temperature from its minimum value is very nominal.

Assume a 20-db amplifier gain. Figure 1 shows (see Appendix A) that a choice of $R_L/R_s = 0.8$ (that actually employed) demands a regenerative gain of about 13 db for the

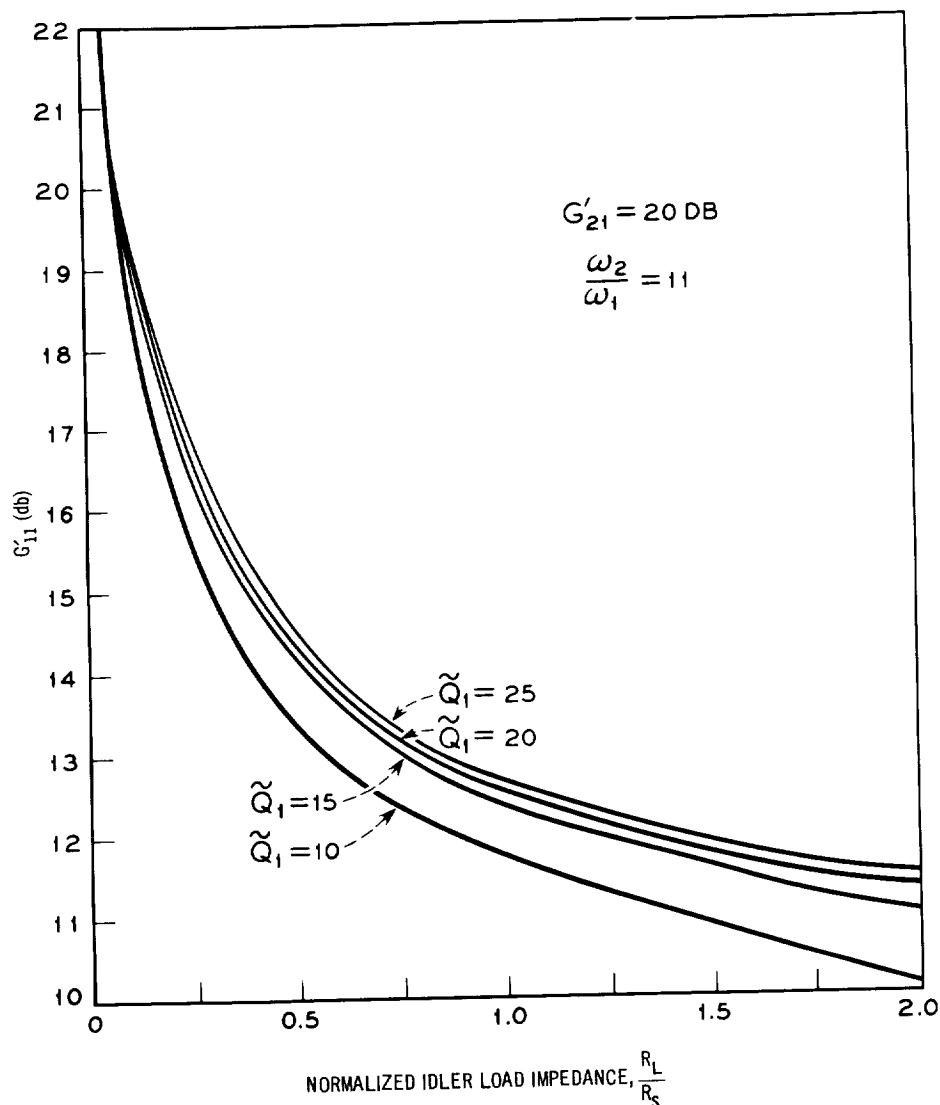


Figure 1 - Regenerative gain G'_{11} of lower-sideband up-converter, which is needed to achieve a total gain G'_{21} of 20 db, vs. normalized idler load impedance R_L/R_s . The regenerative gains are shown for four different dynamic quality factors $Q_1 = 25, 20, 15$, and 10.

lower-sideband amplifier, almost independently of \tilde{Q} ,* in contrast to 20 db for the reflection type. Furthermore (Figure 2), the lower-sideband noise figure (see Appendix B) at $R_L/R_s = 0.8$, at which it is trivially different from its minimum value and is significantly less than that for the reflection amplifier for $\tilde{Q}_1 < 25$. The parameters chosen in evaluating the curves of Figures 1 and 2 are somewhat arbitrary, but are, nevertheless, typical for amplifiers of the type under consideration.

*The quality \tilde{Q} is defined in the next section. It is a quality factor modified by the dynamic capacity ratio; R_L is the load impedance; and R_s is the diode-spreading resistance.

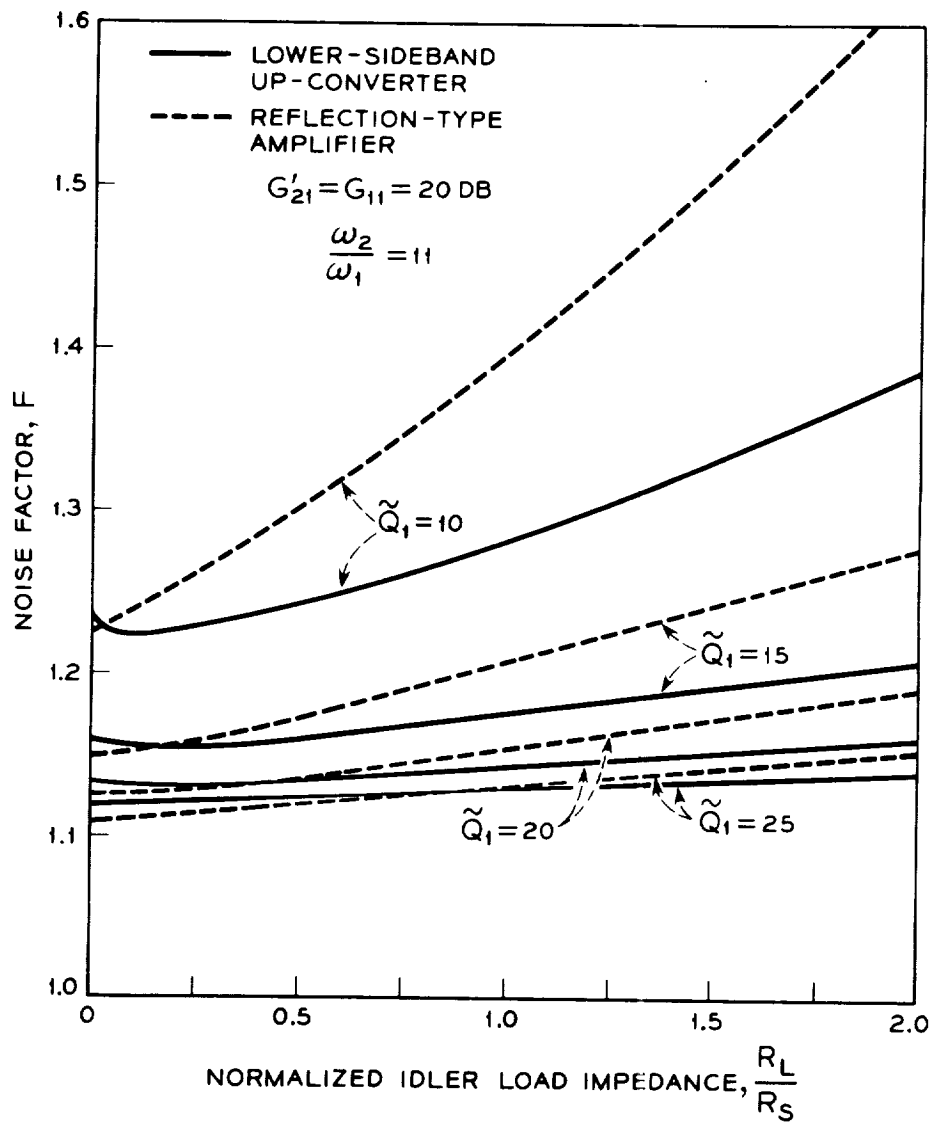


Figure 2 — Noise figures of lower-sideband up-converter and reflection-type amplifiers vs. normalized idler load resistance R_L/R_S . Gains are assumed to be the same at 20 db for both amplifiers

It was under the force of the above arguments, considering both stability and noise, that the lower-sideband amplifier was chosen in preference to the reflection type.

VARACTOR DIODE SELECTION

There are two major requirements in the choice of a varactor diode:

- (1) An impedance sensible in magnitude to that of the generator;

(2) A high Q relative to its variable capacity portion.

The first requirement follows from the desire both to achieve reasonable bandwidth and to avoid large tuning losses in obtaining adequate interaction with the diode. The second follows from the fact that a static capacitor, however excellent its Q , cannot provide a basis for amplification, and its variability must be taken into account. A more realistic definition (Reference 2) is given by

$$\bar{Q} = \frac{Q}{\frac{2C_0}{C_1} - \frac{C_1}{2C_0}}, \quad (5)$$

where C_0 is the static capacitance at the operating point and C_1 is the capacitance associated with the first dynamic term of the Fourier expansion of the time variable charge. The quantity Q is that value conventionally defined for the static capacitance and is equal to

$$Q = \frac{1}{\omega C_0 R_s}, \quad (6)$$

where ω is the frequency and R_s is the spreading resistance of the diode. This result assumes other loss mechanisms to be negligibly effective, and that assumption is given excellent experimental confirmation (Reference 3).

The choice of the value of C_0 is a most complex one if it is to be done correctly. Corresponding to a value of C_0 and its quality factor, there exist generator and load impedances which minimize noise. Transforming to these impedance levels requires tuning elements which are noise sources. Since the diode, as a waveguide scatterer, is an unknown, from present analytic considerations, the computation of that value of C_0 which minimizes total system noise figure is not now achievable.

The actual selection of the static capacity value is intuitive and empirical. Since the capacitive impedance must be of the same order of magnitude as that of the generator, a first choice of C_0 is

$$C_0 = \frac{1}{\omega_s R_g}. \quad (7)$$

This establishes the range of values of C_0 , and initial experiments are performed to range in on an apparently best value. Experience has demonstrated the choice to be fairly broad once a proper region has been found, since the tuning elements contribute little loss.

The final question in choosing a diode relates to the material of the varactor. Diode development at this time has gone in two distinct directions in terms of the gallium arsenide point-contact and the silicon junction varactor. As of this writing, the gallium arsenide junction diode has shown no superiority over the silicon, although it may yet well do so. Selection was between the aforementioned two.

The gallium arsenide point-contact diode has, to date, shown the highest achieved Q in practice, but has two mitigating features:

- (1) Currently available capacities are of order $0.5 \mu\text{f}$.
- (2) Its long-term reliability is not yet proven.

Under these circumstances it was deemed more advisable to choose a silicon junction varactor with a zero bias capacity of $3.5 \mu\text{f}$ and a cutoff frequency of 70 kMc in contrast to double that frequency for the gallium arsenide diode. Measurement of the silicon varactor showed it to possess a value of \bar{Q} of 22 at 960 Mc, which is quite adequate for good performance.

DESCRIPTION OF THE AMPLIFIER

At the time this project was started, no suitable circulator was available at 960 Mc. Of necessity, then, the lower-sideband up-converter was used. However, even if a circulator had been available, the lower-sideband up-converter would still have been chosen since it provides better stability, less severe compression, and possibly even better overall noise performance, as we have discussed earlier. The main features of the amplifier are listed in Table 2.

The reason for operating the diode at zero bias was to simplify the amplifier cavity structure through the absence of a choking section, thereby improving stability. By doing this, a small sacrifice was made in the noise figure, since the dynamic quality factor decreased about 20 percent, from $\bar{Q} = 22$ to $\bar{Q} = 17$, but the specifications were still met without any difficulty. The output circuit was designed to be near critical coupling under the no-pump condition. This coupling condition was selected to improve the stability of the amplifier and the effects of compression.

One potential difficulty in employing a lower-sideband up-converter is the output frequency modulation due to fluctuation in the pump frequency. A slow frequency shift can, of course, be corrected by an AFC system. But fluctuations which are fast and small cannot be corrected satisfactorily. This difficulty is solved by pumping the up-converter of the local frequency generator with the same pump supply as is used for the amplifier. The local oscillator consists of a lower-sideband up-converter and a 931-Mc

Table 2
Main Features of the Lower-Sideband Up-Converter

Feature	Value or Type
Input frequency, f_s	961 Mc
Output (idler) frequency, f_i	10.739 kMc (see Appendix C)
Pump frequency, f_p	11.7 kMc
Pump power supply	TJ klystron (WE 455A) with AFC unit
Bias voltage	0 volt
$\frac{R_L}{R_s}$	≈ 0.8
$\frac{R_g}{R_s}$	≈ 22
Varactor diode	Silicon p-n junction diode in standard coaxial cartridge
f_c	≈ 7.0 kMc
C_0	$\approx 3.5 \mu\mu f$
Estimated noise figure, F	1.19 ± 0.75 db (amplifier alone)

crystal-controlled modulator. The fluctuation of the 30-Mc intermediate frequency is, therefore, exactly the same as that of the crystal oscillator.

A block diagram and photograph of the amplifier are shown in Figures 3 and 4.

CIRCUIT DESIGN

The first-order model of the parametric amplifier is well understood and accounts for the major portion of design. The parameters of this model may be found conveniently from a systematic static or "cold test" procedure (Reference 3), which defines the central operating point and the pump swing. It determines a tuning and impedance-level loading procedure by identifying a static susceptance together with the first-order imaging effects produced by the pumping coupling of signal and image circuits.

The first-order theory oversimplifies the situation somewhat and assumes a sinusoidal pump voltage swing of the diode; it also does not adequately account for the curvature of the charge voltage characteristic of the depletion layer. This latter effect greatly accentuates the charge drawn on the positive portions of the pump swing and increases the capacitive susceptance in viewing the system from either the signal or image port. The actual pump swing is defined in an equilibrium process of the pump, together with

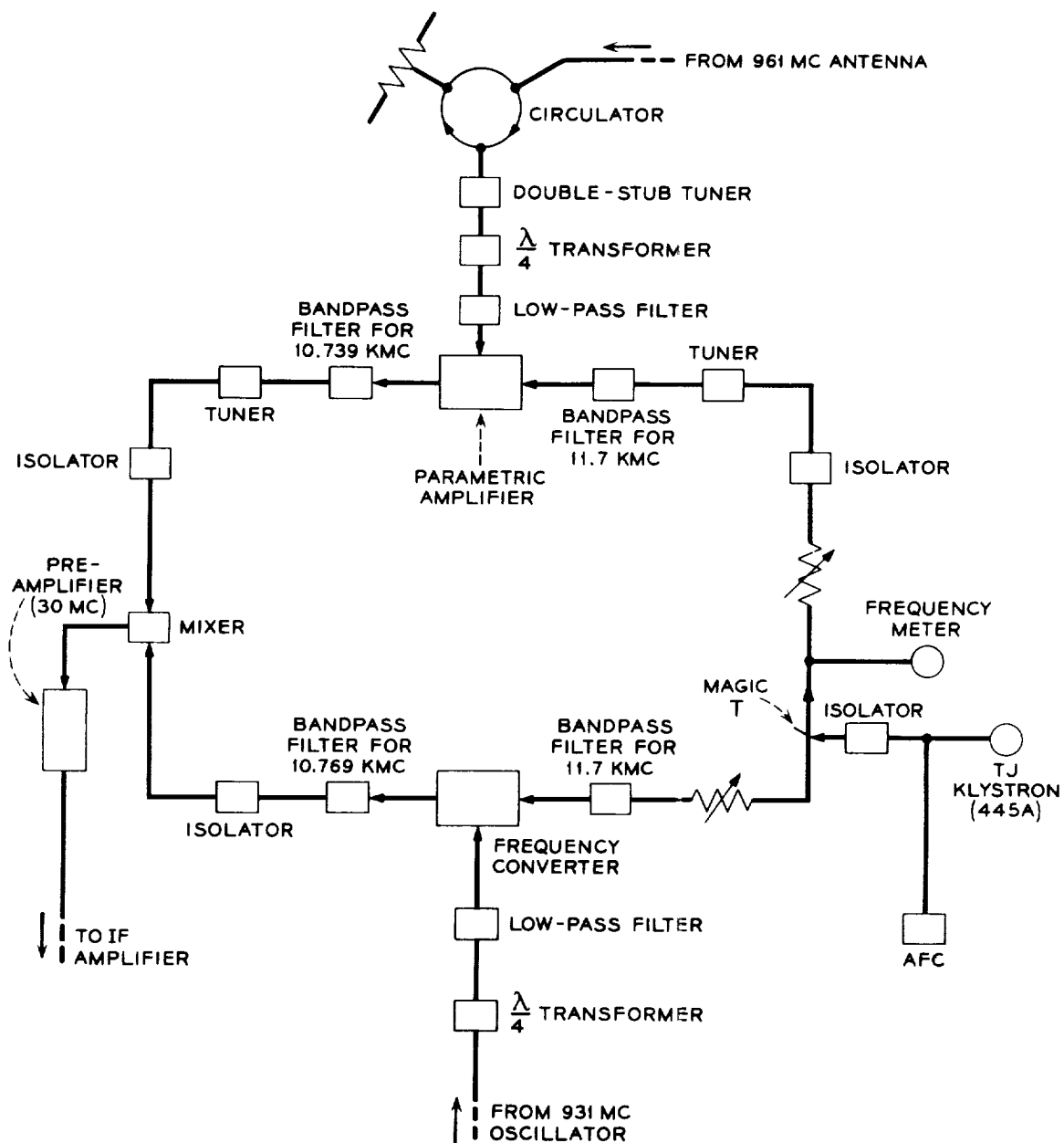


Figure 3 — Block diagram of the 961-Mc amplifier

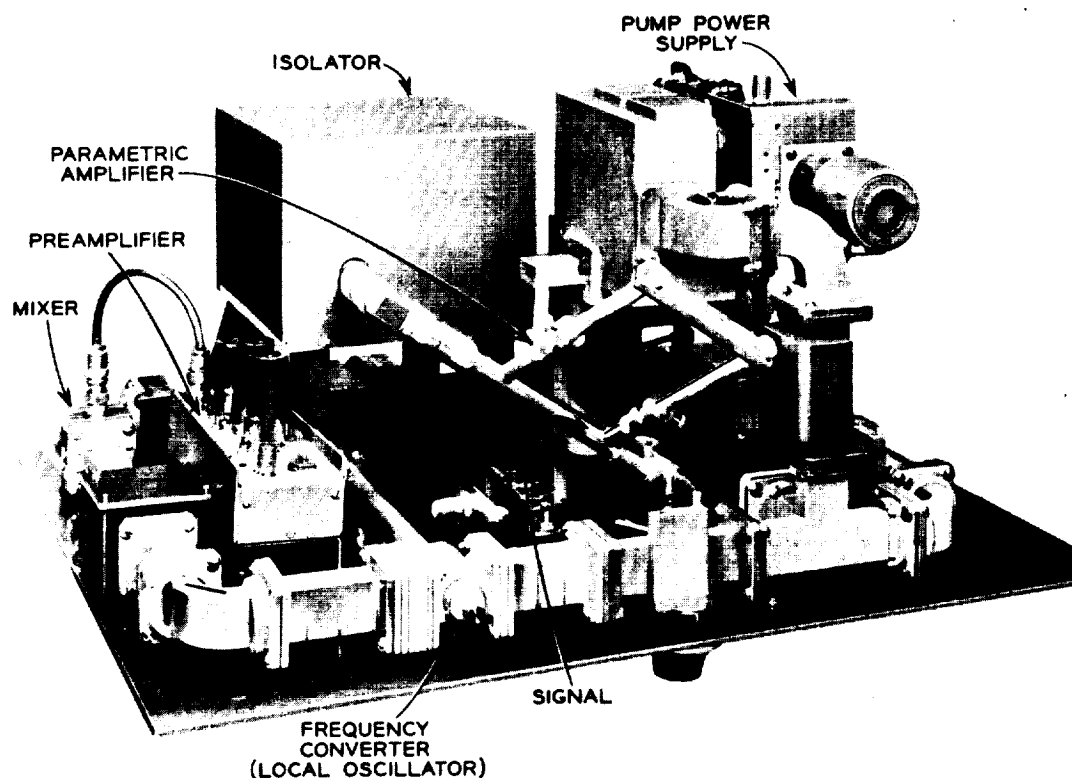


Figure 4 — The 961-Mc amplifier

its complex network as a source, exciting the diode as a nonlinear element. Taking all things into account, the final tuning is completed by a dynamic or "hot" procedure.

The hot tuning can be adequately accomplished by a small bias increase to offset the effects of the pump excursion, and by a tuning touchup in the image circuit. This eliminates the need for variable tuning elements in the signal circuit where the effects of noise are major.

The tuning procedure for minimum noise becomes increasingly difficult with high-Q elements, because the spreading resistance of the diode is small and defies precise determination. Since the spreading resistance is the key to the impedance levels at signal and image frequencies, and since the effects of tuning losses become preponderant, the cold test procedure defers increasingly to the hot test with increasing Q. The demands on Q, however, are modest for most applications which do not demand ultimate noise performance, and the cold test procedure produces a good first design.

Tuning elements generally take the form of quarter-wave transformers, capacitive disks, and stubs. Final touchup is done by adjustable stub tuners in initial development. Figures 3 and 4 show the early circuit for the Echo radar and indicate the use of double stub tuners, but these were replaced by fixed elements in the final development.

The isolator is a most important unit in a stable parametric amplifier. It is desirable that the parametric amplifier system possess a large return loss. Since this number is typically 20 db, and there is a return gain of the order of 13 db produced by regeneration, the isolator must possess a reverse loss greater than 33 db, together with a minimal forward loss. The isolator must be stable to environmental cycling so that its characteristics remain fixed. Small changes in the isolator are considerably swelled by the negative characteristics of the amplifier, and major consideration must be given both to mechanical and electrical stabilization.

AMPLIFIER PERFORMANCE

The amplifier for Project Echo was designed as described in the previous sections. The general performance was satisfactory and closely approximated the design. The best measured overall noise figure, including effects of the isolator, mixer, and IF amplifier, was 1.52 db with 22 db of gain and 20 Mc of bandwidth; overall noise figures of less than 1.6 db were obtainable without any difficulty. The noise figure of the amplifier portion itself was 1.33 db, of which 0.30 db was due to the insertion loss of the isolator, providing an inherent value of 1.03 db. This result is about 0.3 db above the theoretical value for the diode, and it suggests that the excess noise is probably caused by the tuner and the diode mount, and could be further reduced by careful engineering.

The stability of the amplifier was excellent. The gain varied by less than ± 1 db when the input impedance was varied from open-circuit to short-circuit. Under constant-temperature conditions, fluctuations in gain were mainly due to fluctuation in both pump frequency and pump power. The frequency problem was solved by using the electromechanical AFC, which was originally designed for the TJ radio relay system (Reference 4), while the TJ klystron itself was designed for extreme power stability. The electromechanical feature of the AFC tunes the klystron cavity without changing the output power, so that the combination of AFC system and TJ klystron greatly improves the stability of the amplifier.

TEMPERATURE DEPENDENCE OF AMPLIFIER GAIN

Experiment yielded relatively large irreversible temperature sensitivity in the initially constructed amplifier. This difficulty was ultimately traced to a resonance isolator at the amplifier input. It was found that the temperature variation: (a) varied the ferrite magnetization, shifting the resonant frequency slightly; (b) created a minor hysteresis loop in the magnetic structure, not returning the isolator to its initial state. A circulator* was found to be somewhat less critically affected by temperature since, by its essential construction, it is not constrained to operate in the region of a narrow line width.

Since the environmental variations seemed to be beyond passive compensation, it was decided to thermostat the entire amplifier system at $110 \pm 5^\circ\text{F}$ and the circulator at $120 \pm 2^\circ\text{F}$. The system incorporating this thermostating has demonstrated constant performance over a temperature range of 80°F .

Aside from the temperature sensitivity of the circulator, it was of interest to measure the temperature sensitivity of the amplifier section itself. Figure 5 shows a 2-db

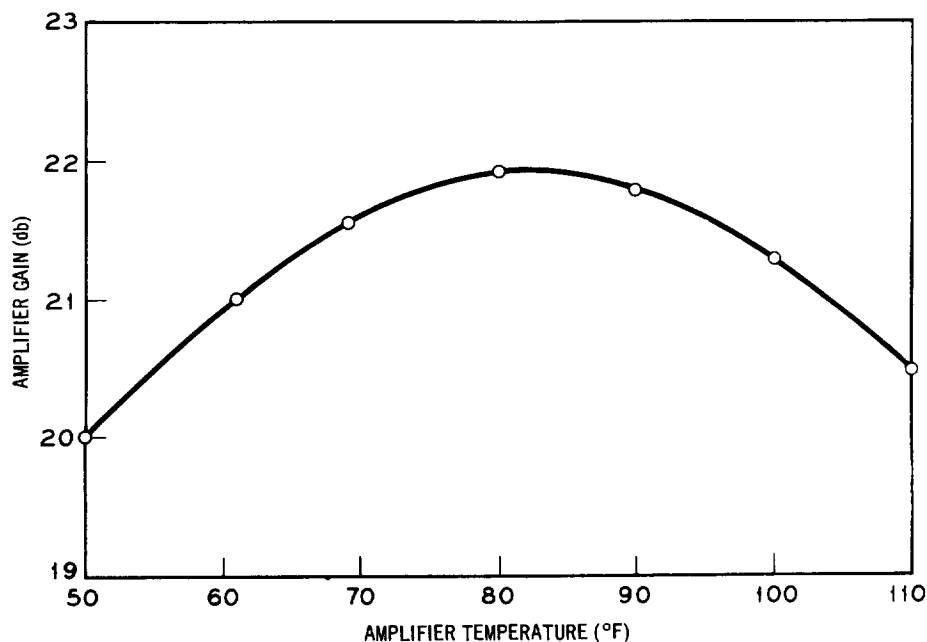


Figure 5 — Amplifier gain vs. amplifier temperature

*Raytheon circulator CLL 8.

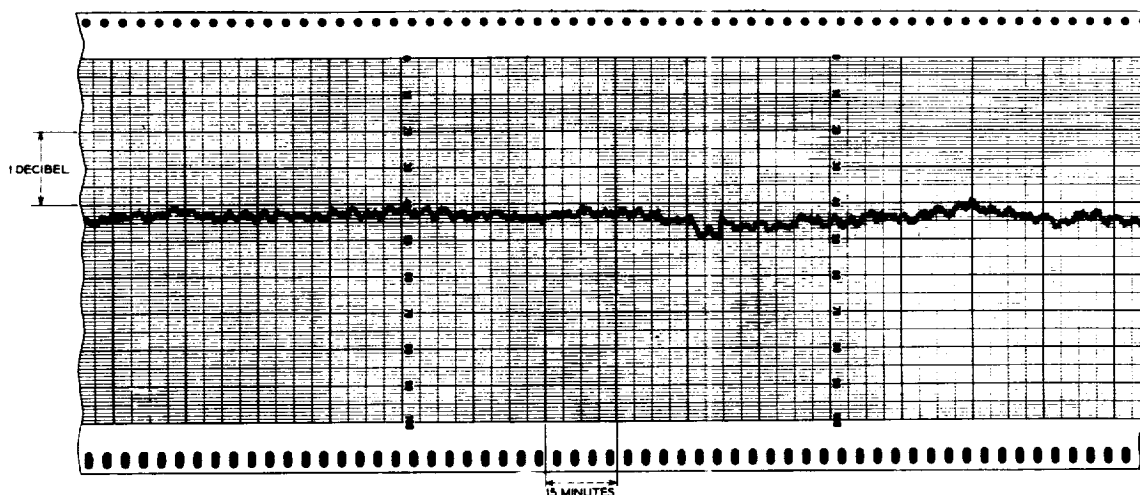


Figure 7 — A part of a trace of the amplifier gain over a period of 24 hours. The gain shift over 24 hours was not more than ± 0.5 db

CONCLUSION

It has been emphasized that careful design and thorough engineering were necessary to obtain a stable low noise amplifier. An overall system noise figure of less than 1.6 db was obtained, with a center-band gain of 22 db and a bandwidth of 20 Mc. The amplifier is very stable, and the gain varies by less than ± 1 db as the input impedance is varied from open-circuit to short-circuit. With an input power of -26 dbm, the gain was 1 db less than the small signal gain of 22 db.

ACKNOWLEDGMENTS

The design of this amplifier is very similar to that of an 860-Mc amplifier designed at Bell Telephone Laboratories for a tropospheric communication system. The authors are greatly indebted to the various members who participated in this 860-Mc project for transmitting to them their valuable experience.

P. J. Pantano did much of the mechanical design and many of the measurements. L. E. Cheesman helped this project during its early period. E. G. Spencer and W. A. Dean supplied the isolators and spent much effort studying the problem of the temperature instability of the isolator characteristics.

Acknowledgement is also made to K. D. Bowers for his helpful criticism and for his encouragement in the study of this project.

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Appendix A

Normalized Generator Impedance for Given Gain and Diode

The gain G_{11} of the reflection-type amplifier is given by the square of the voltage reflection coefficient¹ at the signal port, and is equal to

$$G_{11} = \frac{\left| \frac{Z_{11}^*}{R_s} - 1 + \frac{\tilde{Q}_1 \tilde{Q}_2}{1 + \frac{Z_{22}^*}{R_s}} \right|^2}{\left| 1 + \frac{Z_{11}}{R_s} - \frac{\tilde{Q}_1 \tilde{Q}_2}{1 + \frac{Z_{22}^*}{R_s}} \right|^2} \quad (A1)$$

The gain² G_{21}' of the lower-sideband up-converter³ is

$$G_{21}' = \frac{4 \frac{R_g'}{R_s} \frac{R_L'}{R_s} \tilde{Q}_1^2}{\left| \left(1 + \frac{Z_{11}'}{R_s} \right) \left(1 + \frac{Z_{22}'}{R_s} \right) - \tilde{Q}_1 \tilde{Q}_2 \right|^2} \quad (A2)$$

Here Z_{11} and Z_{22} are the input circuit impedance and the idler circuit impedance, respectively, including the static capacitance of the diode; R_g , R_L , and R_s are the generator resistance, idler load resistance, and series resistance of the diode, respectively. At the center frequency Equations A1 and A2 can be simplified to

$$G_{11} = \frac{\left[\left(\frac{R_g}{R_s} - 1 \right) \left(1 + \frac{R_L}{R_s} \right) + \tilde{Q}_1 \tilde{Q}_2 \right]^2}{\left[\left(\frac{R_g}{R_s} + 1 \right) \left(1 + \frac{R_L}{R_s} \right) - \tilde{Q}_1 \tilde{Q}_2 \right]^2} \quad (A3)$$

and

$$G_{21}' = \frac{4 \frac{R_g'}{R_s} \frac{R_L'}{R_s} \tilde{Q}_1 \tilde{Q}_2 \frac{\omega_2}{\omega_1}}{\left[\left(\frac{R_g'}{R_s} + 1 \right) \left(1 + \frac{R_L'}{R_s} \right) - \tilde{Q}_1 \tilde{Q}_2 \right]^2} \quad (A4)$$

¹Uenohara, M., "Noise Consideration of the Variable-Capacitance Parametric Amplifier," Proc. I.R.E. 48(2):169-179, February, 1960.

²The notation associated with a lower-sideband up-converter is primed to indicate that the scattering properties of the network are not taken simultaneously with those of the reflection type, whose quantities are left unprimed.

³Kurokawa, K., and Uenohara, M., "Minimum Noise Figure of the Variable-Capacitance Amplifier," Bell System Tech. J. 40(3):695-722, May, 1961.

where ω_1 and ω_2 are the angular frequencies of the input and idler, respectively, and the circuit losses are assumed to be negligible. Fluctuation in pump power alters \tilde{Q}_1 and \tilde{Q}_2 and, if the gain is reasonably high, fluctuation in $\tilde{Q}_1\tilde{Q}_2$ is equivalent to fluctuation in the circuit impedances R_g and R_L . At the same time, any fluctuation in the pump frequency adds reactive components to the circuit impedances. If the bandwidth of the amplifier is reasonably broad, a small fluctuation in the pump frequency does not affect the gain of the amplifier significantly. The gain-bandwidth product is a simple measure of the stability of the amplifier with respect to fluctuation in the pump frequency.

The most relevant measure of the stability of a parametric amplifier is the sensitivity of its gain to fluctuations in the resistance of the circuits. We shall now examine a measure of this sensitivity with the amplifier gain set at 20 db, a value close to that used in practice. The value of R_s/R_g is not coincident for both amplifier types in this case. The major gain instability due to an impedance variation is caused by the denominators in Equations A3 and A4, which are proportional to the regenerative gains of the amplifiers. To compare relative stability, therefore, it is much easier to compare the gain G_{11} of the reflection-type amplifier with the regenerative part G_{11}' (or input circuit gain) of the lower-sideband up-converter, whose total gain G_{21}' is equal to G_{11} . If G_{11}' is lower than G_{11} , the lower-sideband up-converter is the more stable, and vice versa. The values of R_g/R_s , which provide gains G_{11} and G_{21}' for a given \tilde{Q}_1 and \tilde{Q}_2 are calculated from the following equations:

$$\begin{aligned} \frac{R_g}{R_s} &= \left(\frac{\tilde{Q}_1\tilde{Q}_2}{1 + \frac{R_L}{R_s}} - 1 \right) \left[\frac{G_{11} + 1}{G_{11} - 1} + \sqrt{\left(\frac{G_{11} + 1}{G_{11} - 1} \right)^2 - 1} \right], \\ &= \left(\frac{\tilde{Q}_1\tilde{Q}_2}{1 + \frac{R_L}{R_s}} - 1 \right) \left[\frac{(\sqrt{G_{11} + 1})^2}{G_{11} - 1} \right], \end{aligned} \quad (A5)$$

for the reflection-type amplifier, and

$$\begin{aligned} \frac{R_g'}{R_s} &= \left[\frac{2 \frac{R_L'}{R_s} \tilde{Q}_1^2}{\left(1 + \frac{R_L'}{R_s} \right)^2 G_{21}} + \frac{\tilde{Q}_1\tilde{Q}_2}{1 + \frac{R_L'}{R_s}} - 1 \right] \\ &+ \sqrt{\left[\frac{2 \frac{R_L'}{R_s} \tilde{Q}_1^2}{\left(1 + \frac{R_L'}{R_s} \right)^2 G_{21}} + \frac{\tilde{Q}_1\tilde{Q}_2}{1 + \frac{R_L'}{R_s}} - 1 \right]^2 - \left(\frac{\tilde{Q}_1\tilde{Q}_2}{1 + \frac{R_L'}{R_s}} - 1 \right)^2} \end{aligned} \quad (A6)$$

for the lower-sideband up-converter. The values of R_g/R_s which provide 20-db gain for given values of \tilde{Q}_1 and \tilde{Q}_2 are plotted as a function of R_L/R_s in Figure A1. Substituting the appropriate value of R_g/R_s for the lower-sideband up-converter, Equation A4 can be solved for G_{21}' . The results are plotted in Figure 2 in the body of this report.

The load coupling factor for which G_{11} exceeds G_{21}' is determined by the condition

$$\left[\left(\frac{R_g'}{R_s} - 1 \right) \left(1 + \frac{R_L'}{R_s} \right) + \tilde{Q}_1 \tilde{Q}_2 \right]^2 = 4 \frac{R_g'}{R_s} \frac{R_L'}{R_s} \tilde{Q}_1 \tilde{Q}_2 \frac{\omega_2}{\omega_1}. \quad (\text{A7})$$

A high-gain approximation leads to the simpler equation

$$\left(1 + \frac{R_L'}{R_s} \right) \frac{R_g'}{R_s} = \frac{R_L'}{R_s} \left(\frac{R_g'}{R_s} + 1 \right) \frac{\omega_2}{\omega_1}, \quad (\text{A8})$$

from which R_L'/R_s is found to be

$$\begin{aligned} \frac{R_L'}{R_s} &= \frac{\omega_1}{\omega_2 \left(1 + \frac{R_g'}{R_s} \right) - \omega_1} \\ &\approx \frac{\omega_1}{\omega_2 - \omega_1} \quad \text{if} \quad \frac{R_g'}{R_s} \gg 1. \end{aligned} \quad (\text{A9})$$

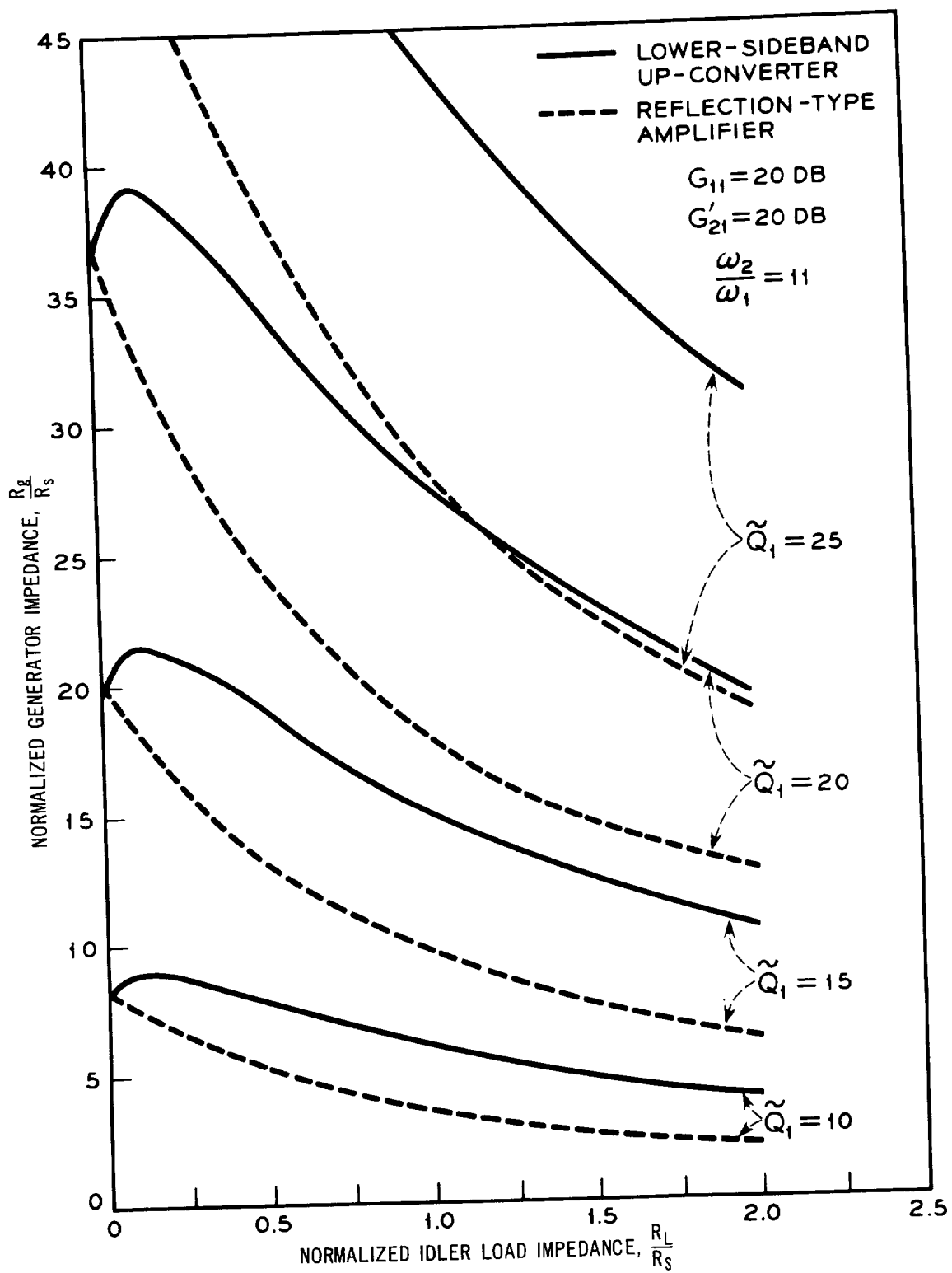


Figure A1 — Normalized generator impedance R_g/R_s needed to obtain the gain of 20 db vs. normalized idler load impedance R_L/R_s .

Optimum Idler Frequency for Arbitrary Idler Load

For high-gain and minimum-noise operation, the ratio of signal frequency to idler frequency is given by Kurokawa and Uenohara¹. However, when R_L is not zero, the optimum idler frequency is different from that obtained using their equation, and it is instead determined by finding the value of ω_2 which minimizes the noise figure given by the following equation:

$$F = 1 + \frac{R_s}{R_g} + \frac{\left(\frac{R_L}{R_s} + 1\right) R_s}{R_g} \frac{\omega_1}{\omega_2} \frac{\left(1 + \frac{R_g}{R_s}\right)^2}{\tilde{Q}_1 \tilde{Q}_2} \quad (C1)$$

$$= \left(1 + \frac{R_s}{R_g}\right) \left(1 + \frac{\omega_1}{\omega_2}\right) + \frac{1}{G_{21}}.$$

Using the high-gain approximation (Equation A6 of Appendix A) for R_g/R_s as a function of frequency, we find

$$F = \left(1 + \frac{\omega_1}{\omega_2}\right) \left[1 + \frac{1}{\tilde{Q}_1^2 \frac{\omega_1}{\omega_2}} \frac{1}{1 + \frac{R_L}{R_s}} \right] + \frac{1}{G_{21}}. \quad (C2)$$

Equation B10 leads readily to the result that

$$\frac{\omega_2}{\omega_1} = \frac{\tilde{Q}_1^2}{\left(1 + \frac{R_L}{R_s}\right) \left(1 + \sqrt{1 + \frac{\tilde{C}_1^2}{1 + \frac{R_L}{R_s}}}\right)}. \quad (C3)$$

The error in approximating the value of R_g/R_s at high gain is usually small and provides negligible error in determining noise figure. Letting $R_L/R_s = 0.8$ and $\tilde{Q}_1 = 17$, we find that $\omega_2/\omega_1 = 11.7$. The optimum idler frequency is thus about 11.3 kMc. From this idler frequency the pump frequency is determined to be 12.261 kMc. Since the optimum frequency result leads to broad noise figure minima, it was decided to forego

¹Kurokawa, K. and Uenohara, M., "Minimum Noise Figure of the Variable-Capacitance Amplifier," Bell System Tech. J. 40(3):695-722, May 1961.

the exact pump frequency calculated in favor of one within the band of the Western Electric 445A klystron. This klystron, which has been designed for the TJ system, has a notably stable and long life. It was operated as a pump at the top of its range, at 11.7 kMc, producing an idler frequency of 10.739 kMc.

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